ON THE TOPOLOGY AND GEOMETRY OF n-NORMED SPACES

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In the series of five lectures, we shall discuss the topology and geometry of n-normed spaces. We begin with the concepts of n-inner products and n-norms for any $n \in \mathbf{N}$, which may be viewed as generalizations of the concepts of inner products and norms. The definition of n-inner products was originally formulated by Misiak [1, 2], while the notion of n-norms was developed by Gähler [3, 4, 5]. We shall discuss some results on n-inner product spaces and n-normed spaces, including the topology and the notion of orthogonality in n-normed spaces. Related to the n-inner product (and its deduced n-norm), we have the Cauchy-Schwarz inequality and accordingly the cosine of the angle between two n-dimensional subspaces intersecting on an $(n-1)$-dimensional subspace. We are then interested in the formula for the angle between two subspaces (which may be of different dimensions) of an inner product space in general, as in [6, 7, 8].

The five lectures will be organized as follows:

(1) Introduction: n-inner product spaces and n-normed spaces.
(2) Topology in n-normed spaces
(3) Orthogonality in n-normed spaces
(4) Angles between two subspaces of a real inner product space - I
(5) Angles between two subspaces of a real inner product space - II

References

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